

Calculation Policy



Making Learning An Adventure

The following calculation policy has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculations across the school. The policy has been devised with members of staff using the White Rose Maths Hub Calculation Policy with further material added and adapted. It is a working document and will be revised and amended as necessary

Age stage expectations: The calculation policy is organised according to age stage expectations as set out in the National Curriculum 2014 and the method(s) shown for each year group should be modelled to the vast majority of pupils.

However, it is vital that pupils are taught according to the pathway that they are currently working at and are showing to have 'mastered' a pathway before moving on to the next one. Of course, pupils who are showing to be secure in a skill can be challenged to the next pathway as necessary.

Choosing a calculation method: Before pupils opt for a written method, they should first consider these steps:

Can I do it in my head using a mental strategy?

Could I use some jottings to help me?

Should I use a formal written method to work it out?

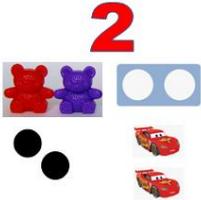
Addition- Reception Early learning goals:

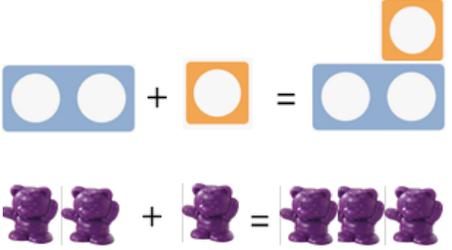
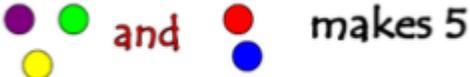
Count reliably with numbers from 1 to 20, place them in order.

Say which number is one more than a given number.

Using quantities and objects, they add two single-digit numbers and count on to find the answer

Key Vocabulary:
 add, more, and make,
 sum, total altogether
 score double one more,
 two more, ten more...
 how many more to

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Recognise numbers up to 20 and understand the meaning of each number by recognising and knowing their clusters</p>	<p>Children use everyday objects and resources to represent each number up to 20. For example:</p> 	<p>Children are shown different visual representations and recognise what number it represents</p> 	<p>Children are shown a digit and understand what this means e.g. 2</p>
<p>Count on in ones and say which number is one more or less than a given number</p>	<p>Children physically move themselves along the numbers e.g. jump or walk</p>  <p>Children use everyday objects, count them out and physically add one more or take one away (one less)</p>	<p>Children use a number line or number track to 20 and count along it forwards or backwards</p>	<p>1,2,3,4,5</p> <p>One more than 2 is 3 $2 + 1 = 3$</p> <p>One less than 4 is 3 $4 - 1 = 3$</p>
<p>Relate addition to combining two groups of</p>	<p>Children physically use concrete resources and manipulatives and add 2 groups together.</p>	<p>Children see or draw a visual representation to add the two groups together</p>	<p>The written form is used $2 + 3 = 5$</p>

<p>objects using practical resources, role play, stories and songs.</p>			
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Addition Year 1 statutory requirements:

Count to and across 100, forwards beginning with 0 or 1, or from any given number.

Given a number, identify one more.

Read, write and interpret mathematical statements involving addition (+), and equals (=) signs.

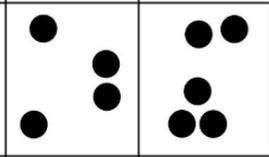
Represent and use number bonds and related subtraction facts within 20

Add one-digit and two-digit numbers to 20, including zero.

- Solve one-step problems that involve addition using concrete objects and pictorial representations, and missing number problems.

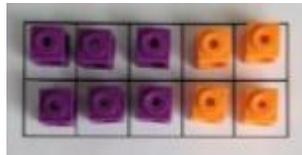
Key Vocabulary:

+, add, more, plus,
make, sum, total
altogether score double,
near double one more,
two more... ten more
how many more to
make... How many

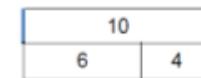
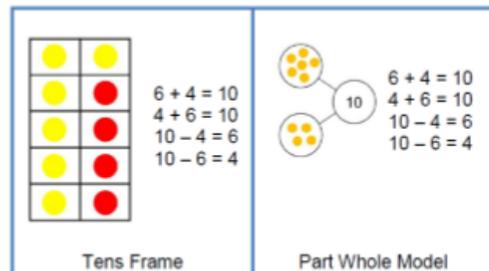
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Identify and represent numbers using objects and pictorial representations (multiple representations)</p>	<p>Children use equipment and everyday objects to make and represent a number</p> 	 <p>Children draw different representations of a number</p>	<p>5</p>

Represent & use number bonds and related subtraction facts within 20

Children use practical equipment on a tens frame to represent the bonds



Children see and draw images in a tens frame and part whole model to find number bonds and related facts



$$6 + 4 = 10$$

$$4 + 6 = 10$$

$$10 - 4 = 6$$

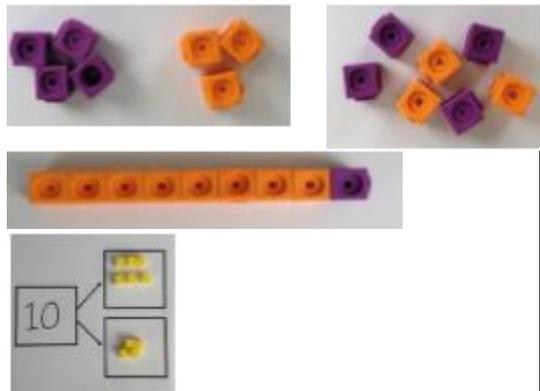
$$10 - 6 = 4$$

Bar Model

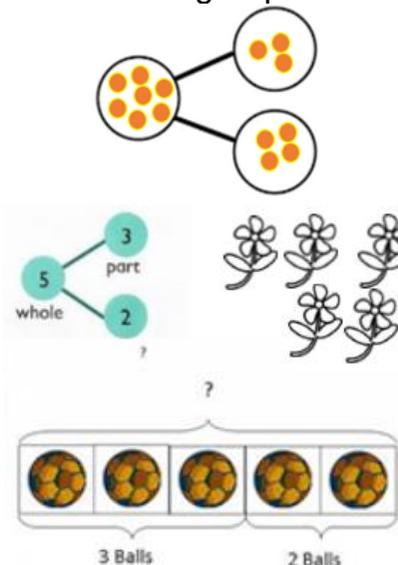
'1 more than 5 is equal to 6.'
 '2 more than 5 is 7.'
 '8 is 3 more than 5.'

Combine two parts to make a whole

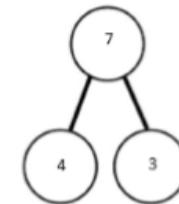
Children will use lots of different resources such as Numicon, counters, eggs, shells, teddy bears and everyday objects



Children will use and draw pictures in a to add together 2 numbers as a group or in a bar



Digits will be used
 $4 + 3 = 7$ (four is a part, 3 is a part and the whole is seven)

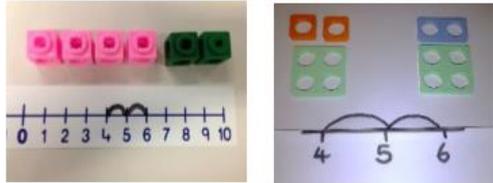
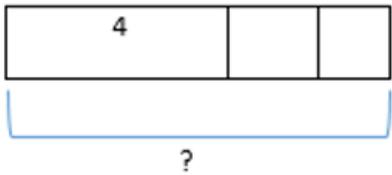
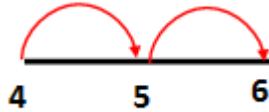
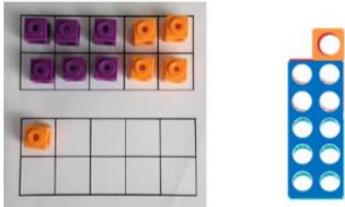
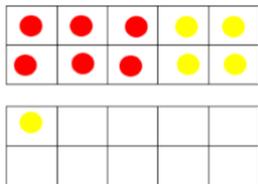
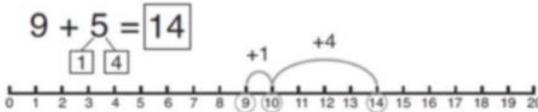


Use concrete resources and a number line to support the addition of numbers. Know

A number line alongside equipment is used

A bar model is used which encourages the children to count on

The abstract number line: What is 2 more than 4? What is the sum of 4 and 4? What's the total of 4 and 2? $4 + 2$

<p>and use strategy of finding the larger number, and counting on in ones from this number</p>			
<p>Regrouping to make 10. This is an essential skill for column addition later.</p>	<p>Use a tens frames and counters/cubes or using Numicon e.g. $6 + 5$</p> 	<p>Children draw the tens frames and counters/cubes</p>  <p>Use pictures or a number line. Regroup or partition one of the numbers e.g.</p> 	<p>Children to develop an understanding of equality e.g.</p> $6 + \square = 11$ $6 + 5 = 5 + \square \quad 6 + 5 = \square + 4$

Addition Year 2 statutory requirements:

- Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts to 100.

Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

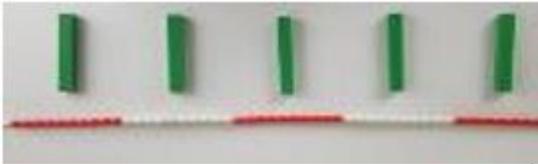
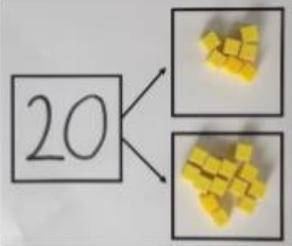
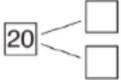
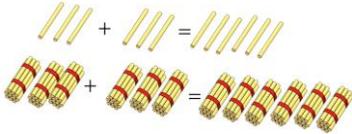
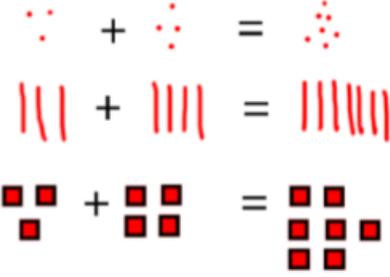
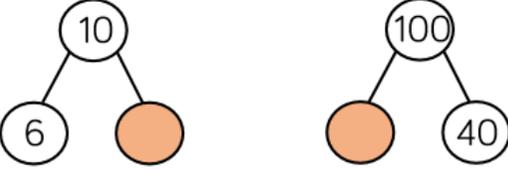
Add numbers using concrete objects, pictorial representations, and mentally, including:

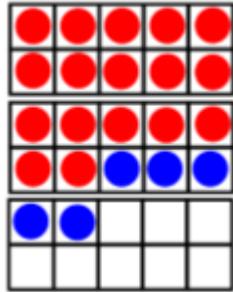
- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers • adding three one-digit numbers.

Solve problems with addition including those involving numbers, quantities and measure

Key Vocabulary:
 +, add, addition, more, plus make, sum, total, altogether, score, double, near double, one more, two more... ten more... one hundred more how many more to make...? How many

Objective &	Concrete	Pictorial	Abstract
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<p>Strategy</p> <p>Add multiples of 10</p>	<p>Model using dienes and bead strings</p> 	<p>Use representations for base ten.</p>  <p>3 tens + 5 tens = ___ tens 30 + 50 =</p>	<p>20 + 30 = 70 = 50 + 20 40 + □ = 60</p>
<p>Use known number facts</p> <p>Part part whole</p>	<p>Children explore ways of making numbers within 20</p> 	<p>Along side of this they use equipment</p>  <p>□ + □ = 20 20 - □ = □ □ + □ = 20 20 - □ = □</p> <p>Also show children calculations where = is at the start e.g. 20 = ? + ?, 20 = ? - ?</p>	<p>□ + 1 = 16 16 - 1 = □ 1 + □ = 16 16 - □ = 1</p>
<p>Using known facts</p>	<p>Use every day items and base ten</p>  <p>□□ + □□ = □□□□ □□□ + □□□ = □□□□□</p>	<p>Children draw representations of H, T and O</p> 	<p>3 + 4 = 7 which leads to 30 + 40 = 70 which leads to 300 + 400 = 700</p> 
<p>Add 2 digit number and ones</p>	<p>17 + 5 = 22 Use ten frame to make 'magic ten'</p>	<p>Use part part whole and number line to model.</p>	<p>Children who are working at a greater depth, will use base ten and alongside this use a formal written method e.g.</p>

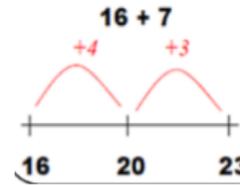
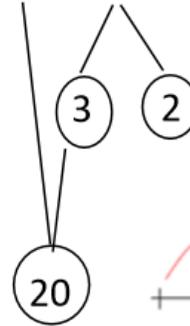


Children explore the pattern.

$$17 + 5 = 22$$

$$27 + 5 = 32$$

$$17 + 5 = 22$$



Tens	Ones
	...
	...

$$\begin{array}{r} 23 \\ + 9 \\ \hline \end{array}$$

Children use a bar model to represent a calculation e.g. $17 + 5 = 22$

22	
17	5

They then explore the related facts

$$17 + 5 = 22$$

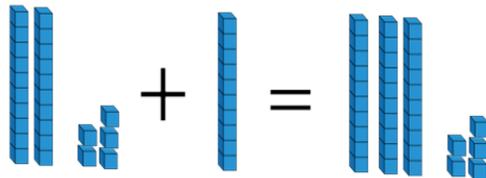
$$5 + 17 = 22$$

$$22 - 17 = 5$$

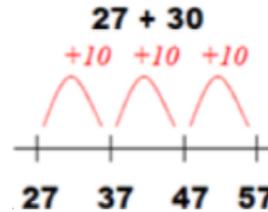
$$22 - 5 = 17$$

Add 2 digit number and ten

Explore that the ones digit don't change



Children draw number lines and add on jumps of ten



$$27 + 10 = 37$$

$$27 + 20 = 47$$

$$27 + \square = 57$$

Children who are working at a greater depth, will use base ten and alongside this use a formal written method e.g.

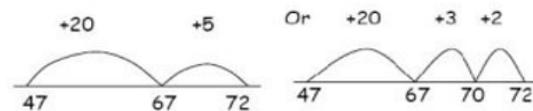
Tens	Ones
	..

$$\begin{array}{r} 23 \\ + 40 \\ \hline \end{array}$$

Add two 2-digit numbers

Model using dienes, place value counters and numicon

Use number line and bridge ten using part whole if necessary. E.g. $47 + 25$



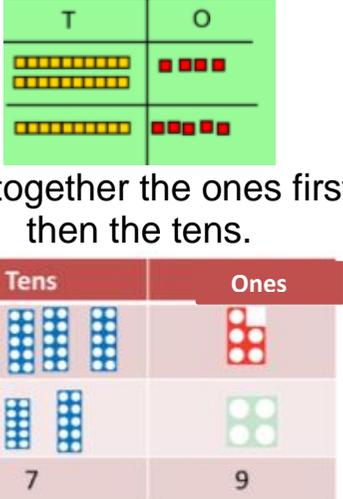
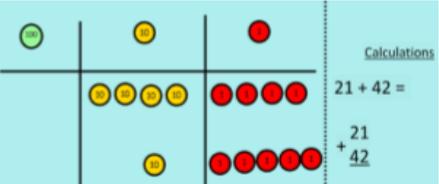
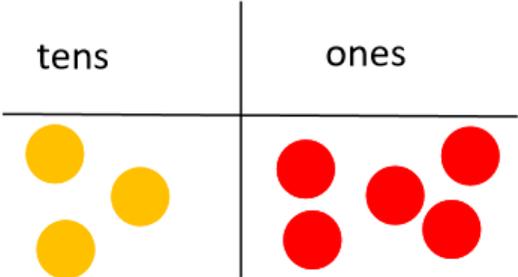
Children who are working at a greater depth, will use base ten and alongside this use a formal written method e.g.

Addition Year 3 statutory requirements:

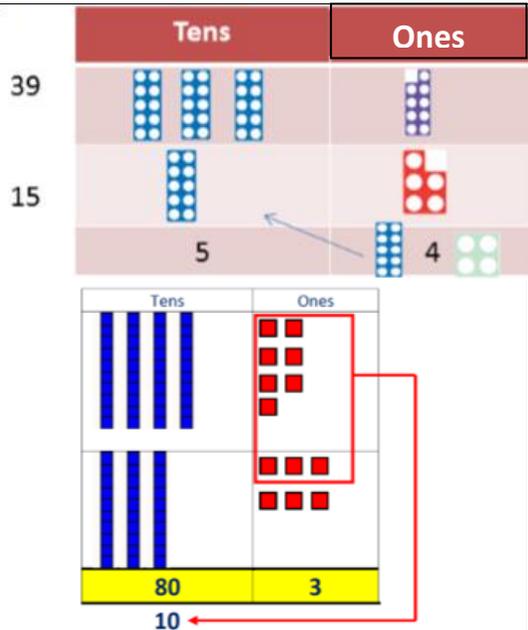
- Find 10 or 100 more than a given number.
- Recognise the place value of each digit in a three-digit number (hundreds, tens, ones).
- Add numbers with up to three digits, using formal written methods of columnar addition

Key Vocabulary:

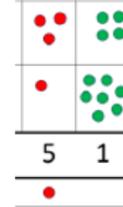
+, add, addition, more, plus
 make, sum, total altogether
 score double, near double one
 more, two more... ten more...
 one hundred more how many
 more to make...? How many

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Column Addition—no regrouping (friendly numbers)</p> <p>Add two or three 2 or 3digit numbers</p>	<p>Model using base ten or numicon</p>  <p>Add together the ones first, then the tens.</p> <p>Move to using place value counters</p> 	<p>Children move to drawing the counters using a tens and one frame</p> 	$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$ <p>Add the ones first, then the tens, then the hundreds</p>

Column Addition with regrouping.



Children draw a representation of the grid to further support their understanding, carrying the ten underneath the line.



Start by partitioning the numbers before formal column to show the exchange

$$\begin{array}{|c|c|c|c|c|c|} \hline 2 & 0 & 5 & & & \\ \hline 4 & 0 & 8 & & & \\ \hline 6 & 0 & + & 1 & 3 & = & 7 & 3 \\ \hline \end{array}$$

Then:

$$\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$$

Addition Year 4 statutory requirements:

Find 1000 more than a given number.

Add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate.

Solve addition two-step problems in contexts, deciding which operations and methods to use and why,

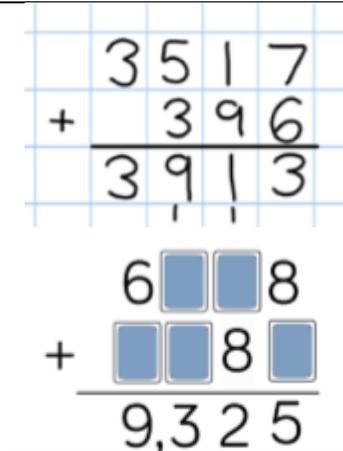
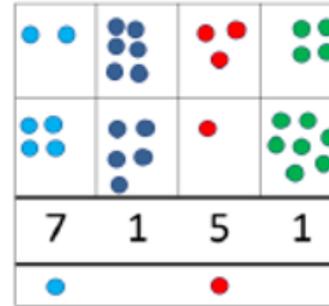
Key Vocabulary:

add, addition, more, plus, increase sum, total, altogether score double, near double how many more to make...?

Consolidate learning from Year 3

Objective & Strategy	Concrete	Pictorial	Abstract
Add numbers with up to 4 digits	Children continue to use base ten or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.	Draw representations using place value counters	Continue from previous work to carry hundreds as well as tens. Relate to money and measures

E.g. 3,242 + 2,213



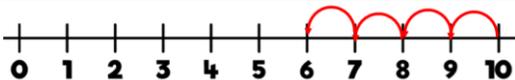
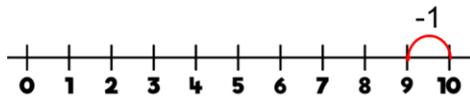
Subtraction - Reception

- Say which number is one less than a given number.
- Using quantities and objects, they subtract two single-digit numbers and count back to find the answer.

Key Vocabulary:

take (away), leave, how many are left/left over?
 How many have gone? one less, two less... ten less...
 How many fewer is...

Objective & Strategy	Concrete	Pictorial	Abstract
Count backwards in familiar contexts such as number rhymes or stories	 	Children draw the items themselves as they decrease	Children see the numbers represented 10,9,8,7,6,5,4,3,2,1
Relate subtraction to 'taking away' using concrete objects and role play		Children count back along a number line to take away alongside equipment 	Children will be shown the calculation which will be read out loud $10 - 6 = ?$

			
<p>Say which number is one less than a given number using numbers to 20</p>	<p>Use equipment and remove one to find one less</p>  <p>Children use large numbers and move backwards to find one less</p> 	<p>Children count back 1 along a number line</p> 	<p>10 take away 1 is .. 1 less than 8 is $5 - 1 =$</p>

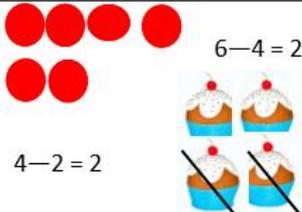
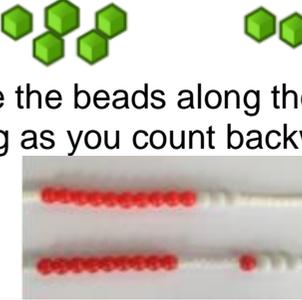
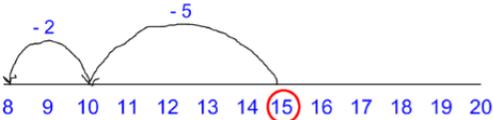
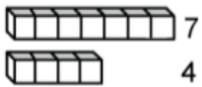
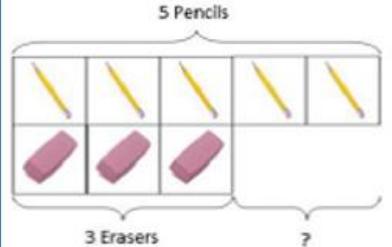
Subtraction Year 1 statutory requirements:

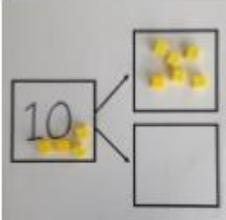
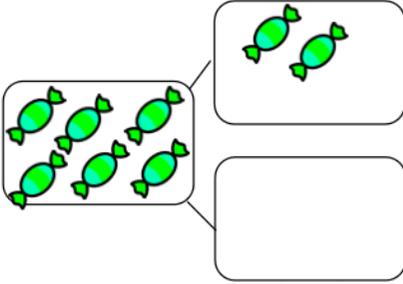
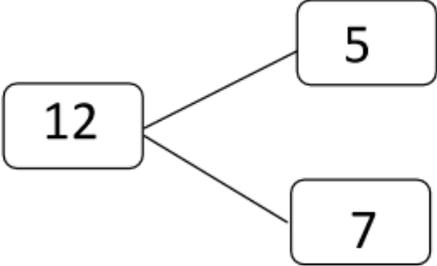
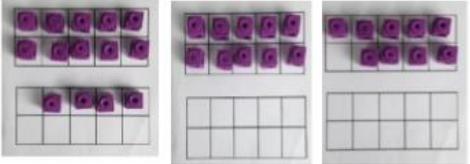
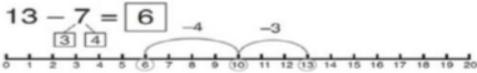
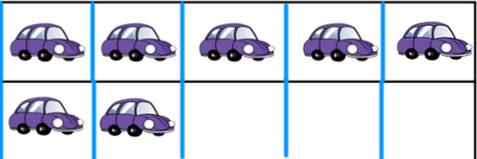
- Say which number is one less than a given number.
- Represent and use number bonds and related subtraction facts within 20.
- Read, write and interpret mathematical statements involving subtraction (-) and equals (=) signs.
 - Subtract one-digit and two-digit numbers to 20, including zero.
- Solve one-step problems that involve subtraction using concrete objects and pictorial representations, and missing number problems

Key Vocabulary:

subtract, take (away), smaller, fewer, minus, less, leave, how many are left/left over? How many have gone? One less, two less, ten less how many fewer

Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away	Cross out drawn objects to show what has been taken away 	$7 - 4 = 3$ $16 - 9 = 7$

	 <p>6-4=2</p> <p>4-2=2</p>		
<p>Counting back</p>	<p>Move objects away from the group, counting backwards</p>  <p>Move the beads along the bead string as you count backwards.</p>	<p>Count back in ones using a number line</p> $15 - 7 = 8$ 	<p>Put 13 in your head, count back 4.</p> <p>What number are you at?</p> $13 - 4 = ?$
<p>Find the difference</p>	<p>Compare objects and amounts</p>  <p>7 'Seven is 3 more than four'</p> <p>4</p> <p>'I am 2 years older than my sister'</p> <p>Lay objects out in a bar model</p>  <p>5 Pencils</p> <p>3 Erasers</p> <p>?</p>	<p>Counting on using a using a number line to find the difference</p> $+ 4$ 	<p>Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister?</p>
<p>Represent and use number bonds and related subtraction facts within 20</p> <p>Part Part Whole model</p>	<p>Link to addition and the Part Part Whole model to model the inverse</p>	<p>Use pictorial representations to show the part.</p>	<p>Move to using numbers within the part whole model</p>

	 <p>If 10 is the whole and 6 is one of the parts, what is the other part? $10 - 6 = 4$</p>				
Make 10	<p>$14 - 9$</p>  <p>Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5</p>	<p>$13 - 7 = 6$</p> <p>Jump back 3 first, then another 4. Use ten as the stopping point.</p> <p>$13 - 7$</p> 	<p>$16 - 8 =$</p> <p>How many do we take off first to get to 10? How many left to take off?</p>		
Bar model	<p>$5 - 2 = 3$</p> 	<p>Children draw their own bar models</p> 	<table border="1" data-bbox="1653 850 2000 914"> <tr> <td>8</td> <td>2</td> </tr> </table> <p>$10 = 8 + 2$ $10 = 2 + 8$ $10 - 2 = 8$ $10 - 8 = 2$</p>	8	2
8	2				

Subtraction Year 2 statutory requirements:

Recall and use subtraction facts to 20 fluently, and derive and use related facts to 100.

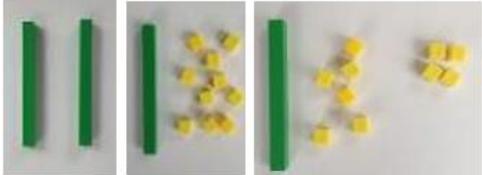
Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

Key Vocabulary:

subtract, subtraction, take (away), minus, leave, how many are left/left over? one less, two less... ten less... one hundred less, How many fewer is... than...? How much less is... ?

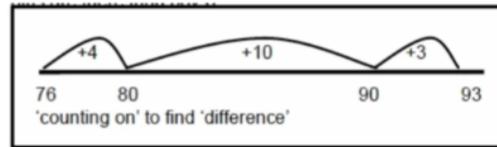
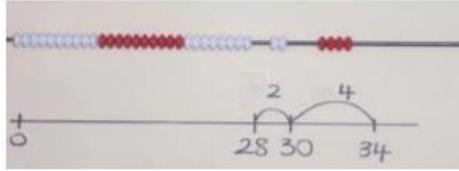
Subtract numbers using concrete objects, pictorial representations, and mentally, including:

- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers

Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	Use a Place Value chart to show how to change a ten into ten ones, use the term 'take and make' E.g. $20 - 4 = 16$ 	$20 - 4 = 16$ 	$20 - 4 = 16$
Partitioning to subtract without regrouping. 'Friendly numbers'	$34 - 13 = 21$ Use base ten to show how to partition the number when subtracting without regrouping 	Children use representations of the base ten and cross off  $43 - 21 = 22$	$\begin{array}{r} 34 \\ 30 \quad 4 \\ -10 \quad -3 \\ \hline 20 \quad 1 \end{array}$ <p>Partition the number 34 into tens and ones. Partition 13 and subtract the ones and the tens. Place the partitioned number back together.</p> <p>They will also be shown this using an expanded column method e.g.</p> $\begin{array}{r} 70 \quad 6 \\ 20 \quad 2 \\ \hline 50 \quad 4 = 54 \end{array}$
Make ten	$34 - 28$ Use a bead bar or bead strings to	Use a number line to count on to next ten and then the rest.	Begin by partitioning into tens and ones.

**strategy
Progression
should be
crossing one
ten, crossing
more than one
ten, crossing the
hundreds.**

model counting to next ten and
the rest.



	60	14	
	70	4	
	20	7	
	40	7	= 47

	6	
	7 ¹	4
	2	7
	<u>4</u>	<u>7</u>

Children working at a greater
depth will also then be shown the
short method

Subtraction Year 3 statutory requirement:

Find 10 or 100 less than a given number.

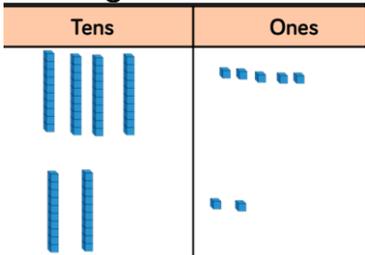
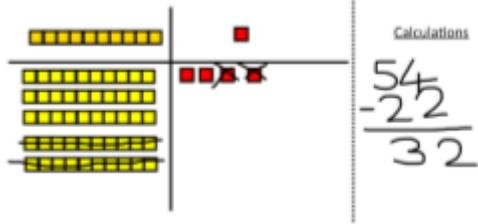
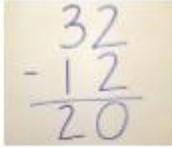
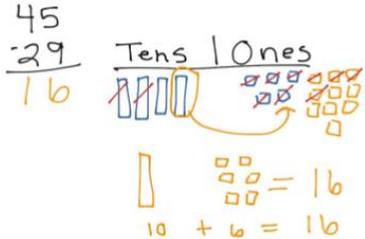
Recognise the place value of each digit in a three-digit number (hundreds, tens, ones).

Subtract numbers with up to three digits, using formal written methods of column subtraction.

Subtract numbers mentally, including: • A three-digit number and ones • A three-digit number and tens • A three-digit number and hundreds.

Key Vocabulary:

subtract, subtraction, take (away), minus leave, how many are left/left over? one less, two less... ten less... one hundred less how many fewer is... than...? how much less is...? difference between half halve

Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping (friendly numbers)	Use base 10 or Numicon to model E.g. $45 - 22 =$ 	Draw representations to support understanding 	$47 - 24 = 23$ $\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$ then 
Column subtraction with regrouping <i>Note: The exchanged ten or hundred is just as important as any other number, therefore, it should be written as clear and as large as any other number, and placed at</i>	Begin with base 10 or Numicon. Move to place value counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.	Children may draw base ten or Place Value counters and cross off. 	Begin by partitioning into place value columns  Then move onto formal method

the top of the column which has been adjusted.

$$\begin{array}{r} 6 \\ \cancel{7}^1 4 \\ 27 \\ \hline 47 \end{array} \quad \begin{array}{r} 4 \\ \cancel{5}^1 \cancel{3} 7 \\ - 254 \\ \hline 283 \end{array}$$

Subtraction Year 4 statutory requirements:

- Find 1000 less than a given number.
- Subtract numbers with up to four digits, using formal written methods of columnar subtraction where appropriate.
- Solve subtraction two-step problems in contexts, deciding which operations and methods to use and why.

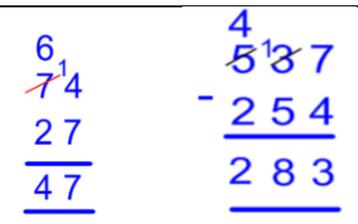
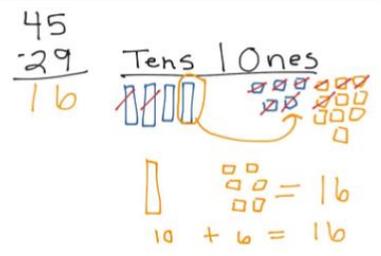
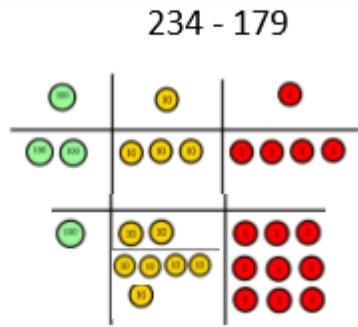
Key Vocabulary:

subtract, subtraction, take (away), minus, decrease leave, how many are left/left over? Difference between half, halve how many more/fewer is... than ? How much more/less

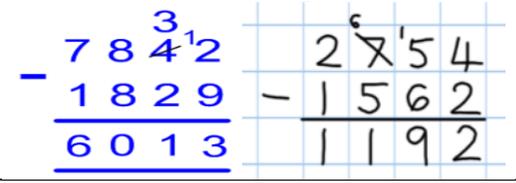
Objective & Strategy	Concrete	Pictorial	Abstract
Subtract with up to 4 digits. Introduce decimal subtraction	Model the process of exchanging using Numicon, base ten and then move to Place value counters	Children may draw base ten or Place Value counters and cross off.	Expanded method $\begin{array}{r} 60 \quad 14 \\ \cancel{7}^1 0 \quad 4 \\ 20 \quad 7 \\ \hline 40 \quad 7 = 47 \end{array} \quad \begin{array}{r} 400 \quad 130 \\ \cancel{5}^1 \cancel{0}^1 \cancel{3}^1 0 \quad 7 \\ 200 \quad 50 \quad 4 \\ \hline 200 \quad 80 \quad 3 = 283 \end{array}$ Then move onto formal short compact method

through context of money

By the end of year 4, pupils should be subtracting numbers up to 4 digits using compact column subtraction method.



Move onto 4 digit numbers



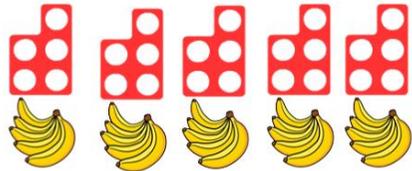
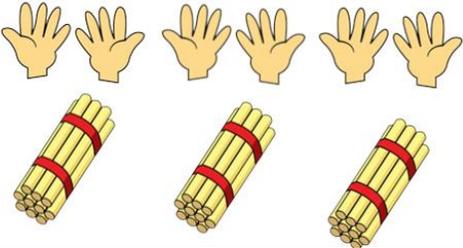
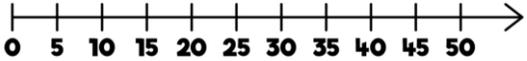
Multiplication Early Learning Goal:

□ They solve problems, including doubling, halving and sharing.

Key Vocabulary:

Double, twice, group, set, 2's, 5's, 10's, multiple

Objective & Strategy	Concrete	Pictorial	Abstract
Use pictorial representations and concrete resources to double numbers to 10.	Use practical activities using manipulatives such as Numicon to double a number 	Draw pictures to show an item has doubled e.g. ladybirds spots 	$2 + 2 = 4$ Double 3 equals 6
Use concrete sources, role play, stories and songs to begin counting in twos, fives and tens.	Use everyday items and objects to count in 2's, 5's and 10's Counting in 2's 	Use a number line alongside the objects Moving on to a numbered number line	0, 2, ?, 6, 8, ?

	<p>Counting in 5's</p>  <p>Counting in 10's</p> 		
--	--	--	--

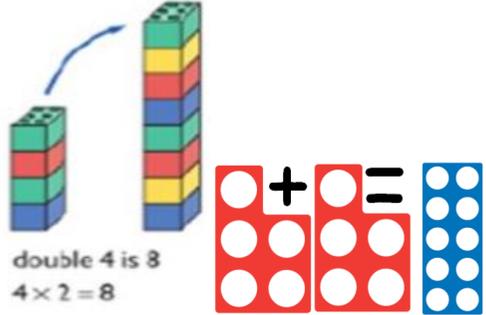
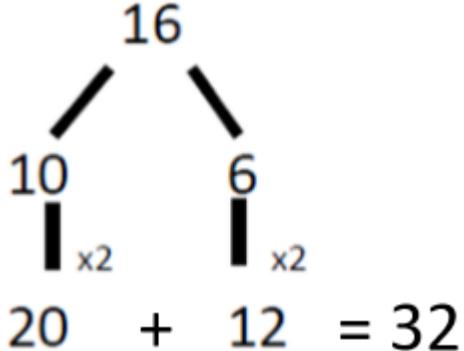
Multiplication

 Year 1 Statutory requirement:

□ Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

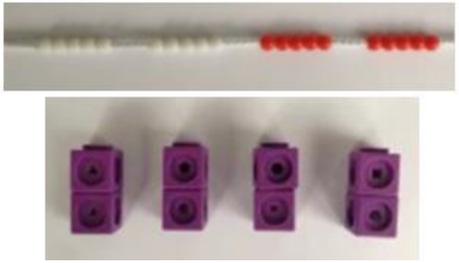
Key Vocabulary:

lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times... .. times as
(big long wide and so

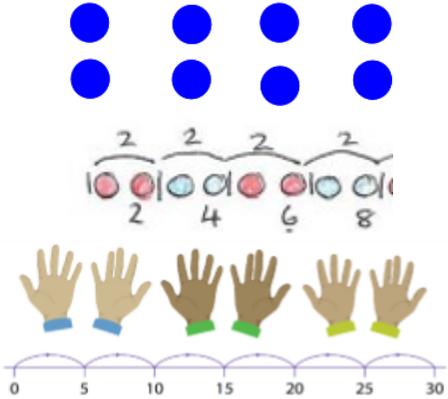
Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	<p>Use practical activities using manipulatives such as Numicon to double and halve a number</p>  <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Draw pictures to show a number has doubled</p> <p>Double 4 is 8</p> 	<p>Partition a number and then double each part before recombining it back together.</p>  <p>$20 + 12 = 32$</p>
Counting in	Count the groups as children are	Children make representations to	Count in multiples of a number

multiples

skip counting, children may use their fingers as they are skip counting.



show counting in multiples.

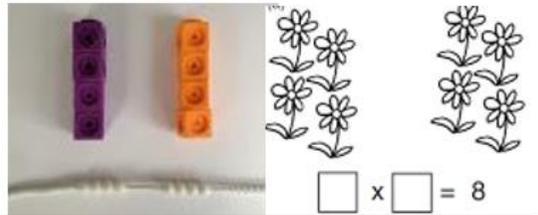


aloud
Write sequences with multiples of numbers.

- 2, 4, 6, 8, 10
- 5, 10, 15, 20, 25, 30

Making equal groups and counting the total

Use manipulatives to create equal groups.



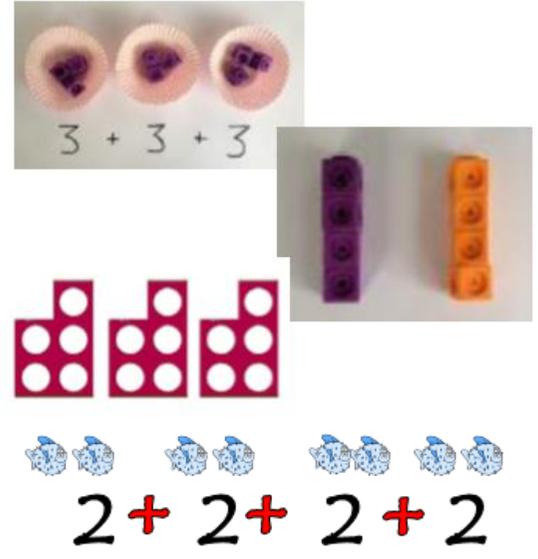
Draw and make representations

Draw  to show $2 \times 3 = 6$

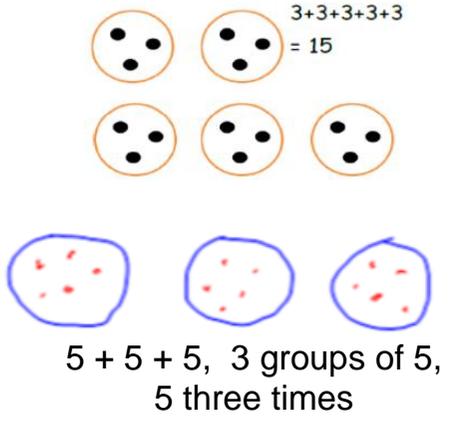
$2 \times 4 = 8$

Repeated addition

Use different objects to add equal groups



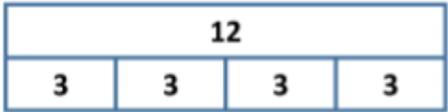
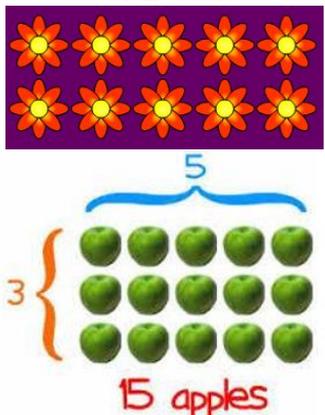
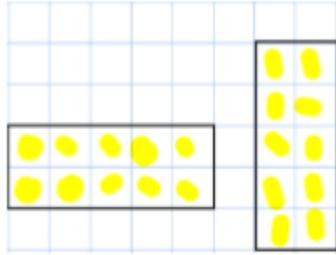
Use pictures and drawings alongside number lines



Move on to a bar model for a more structured approach e.g. $3 + 3 + 3 + 3$,

Write addition sentences to describe the pictures and objects



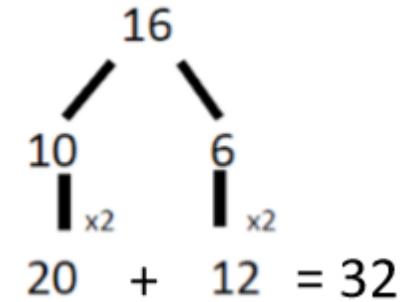
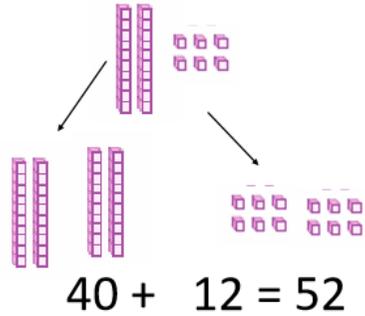
			
Understanding arrays	<p>Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.</p> 	<p>Children draw their own arrays</p>  <p> $2 + 2 + 2 + 2 + 2 = 10$ So 2 five times ($2 \times 5 = 10$) Or $5 + 5 = 10$ So 5 twice ($5 \times 2 = 10$) </p>	<p>Children to be able to use an array to write a range of calculations e.g.</p> $2 \times 5 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $5 \times 2 = 10$ $5 + 5 = 10$

Multiplication Year 2 statutory requirement:

- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
 - Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals (=) signs.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
 - Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

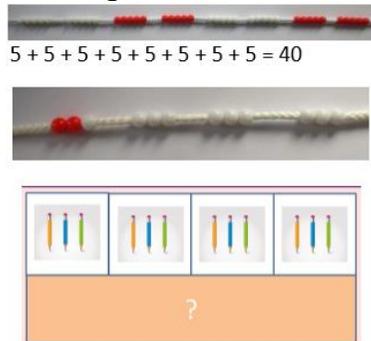
Key Vocabulary:
 lots of, groups of, \times , times, multiply, multiplied by multiple of once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition array

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using base ten and place value counters E.g. double 26	Draw pictures and representations to show how to double numbers	Partition each number and then double each part before recombining it back together

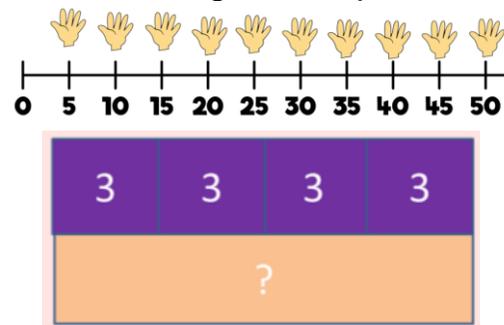


Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)

Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models



Number lines, counting sticks and bar models should be used to show representation of counting in multiples.

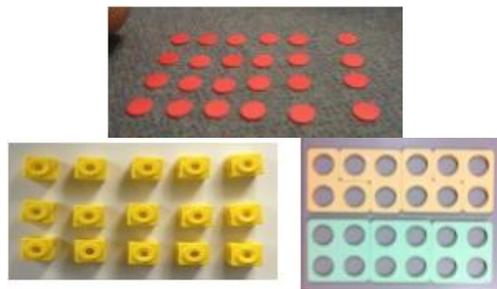


Write sequences with multiples of numbers.

- 0, 2, 4, 6, 8, 10
- 0, 3, 6, 9, 12, 15
- 0, 5, 10, 15, 20, 25, 30
- 1, 3, 5, 7, 9, 11
- 1, 6, 11, 16, 21

Multiplication is commutative

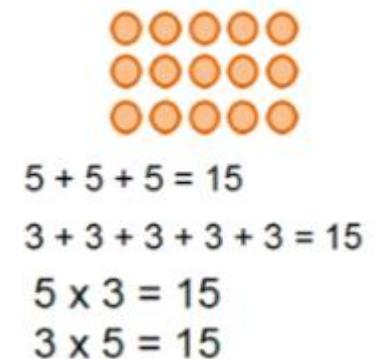
Create arrays using counters, cubes and Numicon



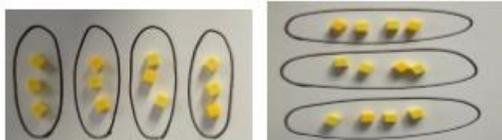
Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication

Children draw their own arrays 3×4 (3 four times)

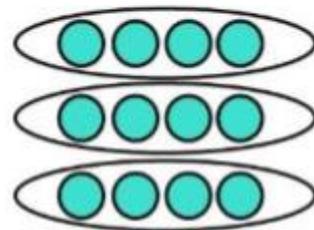
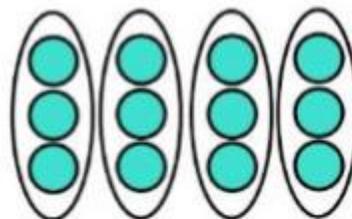
Children to be able to use an array to write a range of calculations e.g.



does not affect the answer.



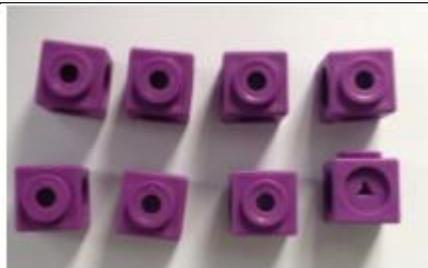
3 x 4 (3 four times) 4 x 3 (4 three times)



4 x 3 (4 three times)

Using the Inverse

This should be taught alongside division, so pupils learn how they work alongside each other.



e.g. $4 \times 2 = 8$ and $2 \times 4 = 8$

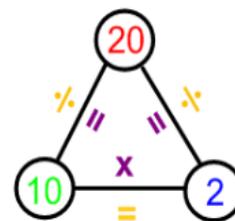
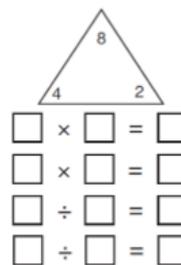
$$8 \div 2 = 4$$

$$8 \div 4 = 2$$

8 divided into groups of 2 = 4

8 divided into groups of 4 = 2

Children draw and complete fact families



$$2 \times 4 = 8$$

$$4 \times 2 = 8$$

$$8 \div 2 = 4$$

$$8 \div 4 = 2$$

$$8 = 2 \times 4$$

$$8 = 4 \times 2$$

$$2 = 8 \div 4$$

$$4 = 8 \div 2$$

Show all 8 related fact family sentences

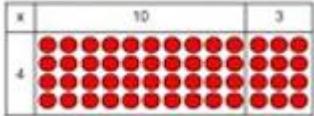
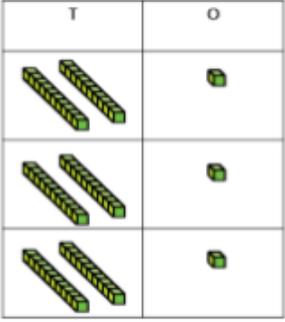
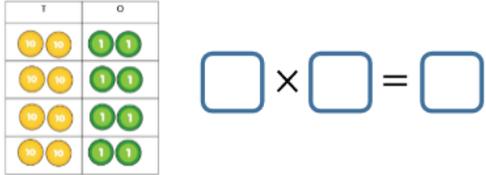
Very important that the children see and use the = sign at the start of a calculation

Multiplication Year 3 statutory requirements:

- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.
- Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
- Solve problems, including missing number problems, involving multiplication including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

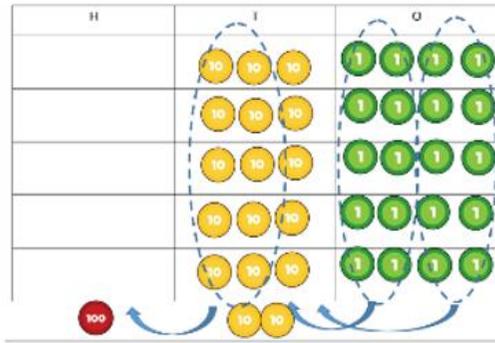
Key Vocabulary:

lots of, groups of \times , times, multiply, multiplication, multiplied by, multiple of, product once, twice, three times... ten times... times as (big, long, wide... and so

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Multiply 2 digit by 1 digit (No exchange)</p>	<p>Show the links with arrays to first introduce the grid method</p>  <p>4 rows of 10, 4 rows of 3 Then move onto base ten and place value counters E.g. 21×3</p>  <p>60 3 = 63</p> <p>E.g. 34×2</p>	<p>Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p> 	<p>What calculation is represented?</p>  <p>$\square \times \square = \square$</p>

Step 5: How many tens and ones do I have altogether? 9 tens add 6 ones = 96

$$5 \times 35$$

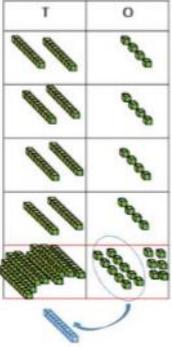
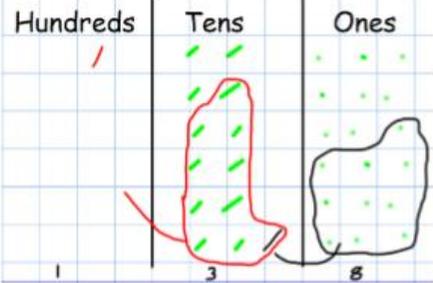


Multiplication Year 4 statutory requirement:

- Recall multiplication and division facts for multiplication tables up to 12×12
- Use place value, known and derived facts to multiply and divide mentally, including: multiply two-digit and three-digit numbers by a one-digit number using formal written layout.
- Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

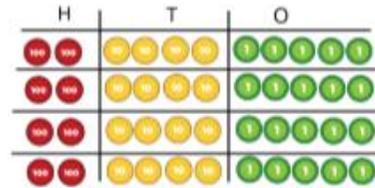
Key Vocabulary:

lots of, groups of times, multiply, multiplication, multiplied by, multiple of, product, factor, once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition

Objective & Strategy	Concrete	Pictorial	Abstract						
<p>Grid method recap from year 3 for 2 digits x 1 digit and move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)</p>	<p>E.g. 24×4 Start with base ten</p>  <p>Step 1: Get 4 lots of 4, 4 lots of 20 Step 2: $4 \times 4 = 16$. Can I make an exchange? Yes I can take ten ones and make a ten Step 3: 4×2 tens plus my extra ten makes 9 Step 4: How many tens do I have? 90 How many ones do I have 6? Step 5: How many tens and ones do I have altogether? 9 tens add 6 ones = 96</p>	<p>Children to represent the counters/base 10, pictorially e.g. the image below</p> 	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" data-bbox="1682 879 2018 978"> <tr> <td>x</td> <td>30</td> <td>5</td> </tr> <tr> <td>7</td> <td>210</td> <td>35</td> </tr> </table> <p>$210 + 35 = 245$</p>	x	30	5	7	210	35
x	30	5							
7	210	35							

Then move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows

E.g. 245×4



Fill each row with 245

Step 1: Get 4 rows of 245 (245 four times)

Step 2: $5 \times 4 = 20$. Can I make an exchange? Yes I can take twenty ones and make 2 tens

Step 3: 4 tens four times plus my extra 2 tens makes 18 tens (180)

Step 4: How many tens do I have? 18. Can I make an exchange? Yes I can exchange 10 tens for 1 hundred which leaves me with 8 tens.

Step 5: How many Hundreds do I have? 8 hundreds plus the extra hundred so 9 hundreds.

Step 6: How many hundreds, tens and ones altogether? $900 + 80 + 0 = 980$

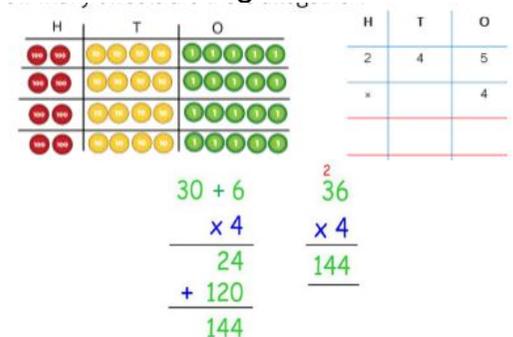
Column Multiplication (TO x O and HTO x O)

It is important at this stage that they always multiply the ones first.

Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping and then moving on to regrouping. $321 \times 2 = 642$

Children to represent the counters/base 10, pictorially e.g. the image below

Two digit number



Leading to a 3 digit number

Hundreds	Tens	Ones

x	300	20	7
4	1200	80	28

Moving on to

327			
x 4			
28			
80			
1200			
1308			

	3	2	7
x			4
<hr/>			
	1	3	0
		1	2

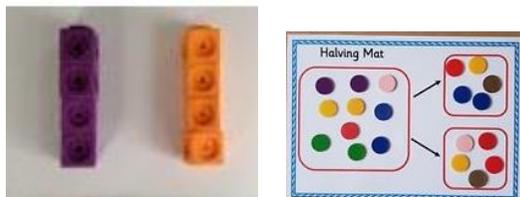
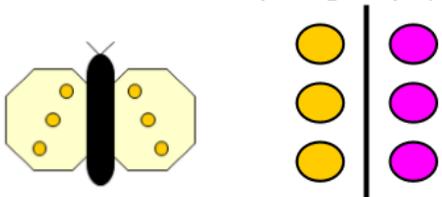
Then

Key Vocabulary:

Share, split, divide, halve, half, groups, lots of

Division Early learning goal statutory requirement:

- They solve problems, including halving and sharing

Objective & Strategy	Concrete	Pictorial	Abstract
Use pictorial representations and concrete resources to halve numbers to 10	<p>Use practical activities using manipulatives such as cubes and Numicon to halve a number</p>  <p>Reinforce the concept of halving through everyday routines such as halving an apple, a cake, piece of bread during snack time.</p>	<p>Children draw representations which show halving (Splitting the amount into 2 equal groups)</p> 	<p>Half of 6 is...</p> <p>I had 10 biscuits and I ate half of them. How many are left?</p>
Share quantities using practical resources, role play, stories and songs.	 <p>Role play example: It is the end of the party and the final two teddies are</p>	<p>Children draw representations which show sharing e.g. in the example below they shared 12 faces into 3 equal groups</p> 	<p>12 shared between 3 people is ...</p>

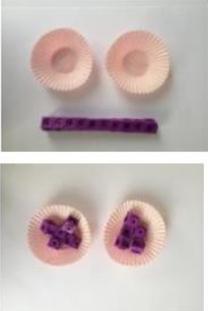
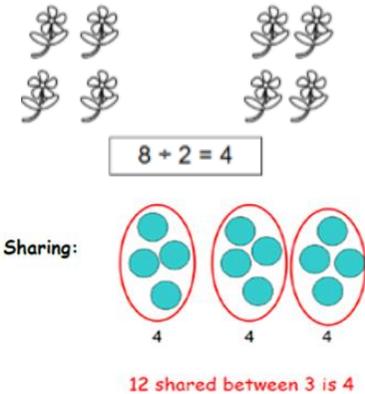
waiting for their party bags. Provide empty party bags and a small collection of items such as gifts, balloons and slices of cake. Ask the children to share the objects between the two bags.

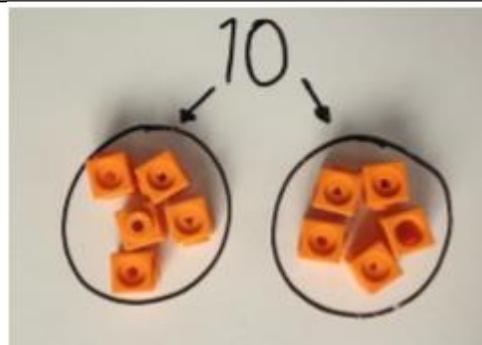
Division Year 1 statutory requirement:

solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Key Vocabulary:

Division, \div , divide, divided by, divided into, left, left over, equal groups, half

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Understand division as sharing using concrete resources.</p>	<p>I have 10 cubes, can you share them evenly between 2 groups?</p> 	<p>Children use pictures or shapes to share quantities.</p>  <p>Pictorial representation of sharing 12 gold coins between 2, 3 and 4</p>	<p>$12 \div 4 = 3$ Share 12 between 4</p>



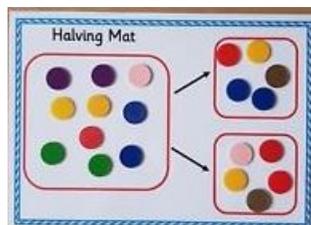
pirates!



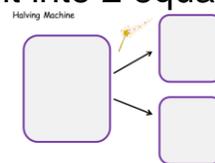
$12 \div 2$ $12 \div 3$ $12 \div 4$

Use pictorial representations and concrete resources to halve numbers

Use practical activities using manipulatives such as cubes and counters to halve a number



Children draw representations and use the halving mat to show halving (Splitting the amount into 2 equal groups)



Half of 12 is....
I had 18 biscuits and I ate half of them. How many are left?

Division Year 2 statutory requirement:

Recall and use division facts for 2, 5 and 10 multiplication tables.

Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (x), division (÷) and equals (=) signs.

Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Find $\frac{1}{3}$; $\frac{1}{4}$; $\frac{2}{4}$; $\frac{3}{4}$ of a length, shape, set of objects or quantity

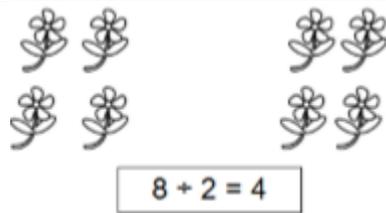
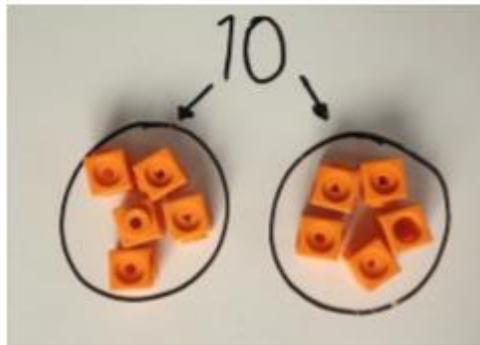
Key Vocabulary:

share, share equally, one each, two each, three each... group, in pairs, threes... tens equal groups of ÷, divide, divided by, divided into

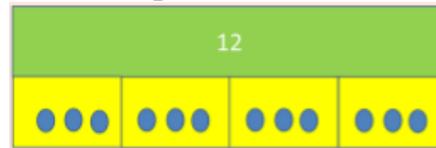
Objective & Strategy	Concrete	Pictorial	Abstract
Understand division as sharing using	I have 10 cubes, can you share them evenly between 2 groups?	Children use pictures or shapes to share quantities.	$12 \div 4 = 3$ Share 12 between 4

concrete resources.

Whilst teaching division, reinforce the connections between fractions and division and rephrase this calculation as $\frac{1}{3}$ of 18 is the same as $18 \div 3 = 6$



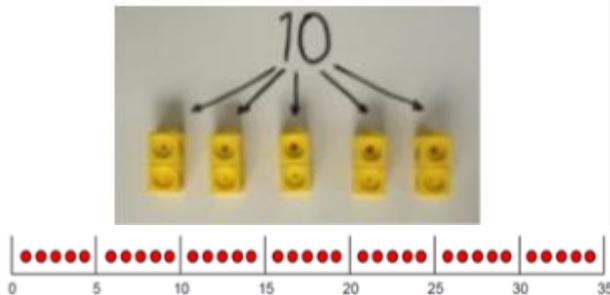
Children use bar modelling to show and support understanding e.g. $12 \div 4 = 3$



Begin to understand division as grouping using concrete resources.

Whilst teaching division, reinforce the connections between fractions and division and rephrase this calculation as $\frac{1}{3}$ of 18 is the same as $18 \div 3 = 6$

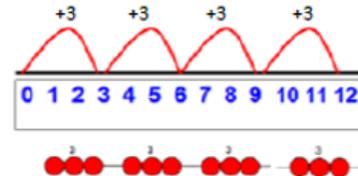
Divide quantities into equal groups e.g. groups of 2
Use cubes, counters, objects or place value counters to aid understanding



12 into groups of 2
 $12 \div 2 = 6$



Use number lines for grouping



$12 \div 3 = 4$

Bar Model - Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group



$20 \div 5 = ?$
 $5 \times ? = 20$

$28 \div 7 = 4$
Divide 28 into 7 groups. How many are in each group?

Division Year 3 statutory requirement:

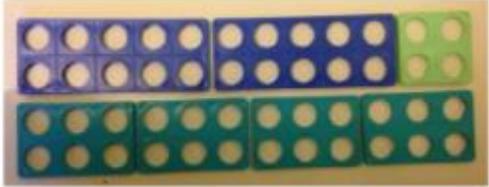
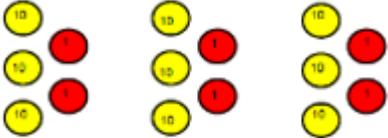
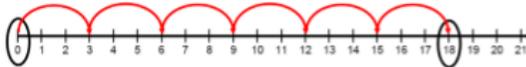
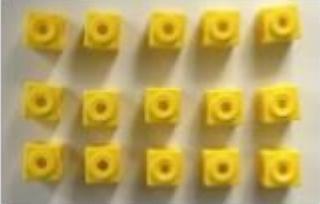
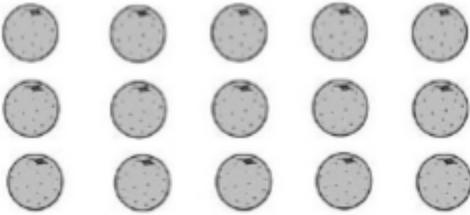
Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables

Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

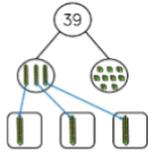
Solve problems, including missing number problems, involving division including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Key Vocabulary:

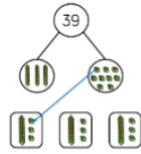
share, share equally, one each, two each, three each... group, in pairs, threes... tens, equal groups of \div , divide, divided by, divided into

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Consolidate understanding of division as <u>grouping</u> using concrete resources.</p>	<p>Use cubes, counters, objects or place value counters to aid understanding.</p>  <p>24 divided into groups of 6 = 4</p> $96 \div 3 = 32$ 	<p>Children use numbered number lines to divide using grouping.</p>  <p>18 into groups of 3 = 6 groups 18 into jumps of 3 = 6 jumps $18 \div 3 = 6$</p> 	<p>How many groups of 6 in 24? $24 \div 6 = 4$</p>
<p>Division with arrays</p>	<p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$</p> 	<p>Draw an array and use lines to split the array into groups to make multiplication and division sentences</p>  <p>E.g. $15 \div 3 = 5$</p>	<p>Find the inverse of multiplication and division sentences by creating eight linking number sentences.</p> $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$
<p>Divide two digit number by one digit with no remainders</p>	<p>Children represent a calculation using base ten and then share the tens and ones e.g. $39 \div 3 = 13$</p>	<p>Children will use a part whole model and draw in the tens and ones themselves</p> <p>They will also be shown how to use a number line:</p>	<p>Children use their division knowledge and calculate the answer to questions like: $96 \div 8$</p> $96 \div 3$ $96 \div 6$

Step 1: Share the tens

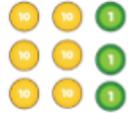


Step 2: Share the ones



Then they move onto place value counters

e.g. $63 \div 3$.



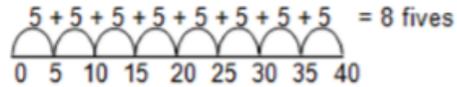
First they make 63 and then share it into 3 rows.

T	O
10 10	1
10 10	1
10 10	1

Example without remainder:

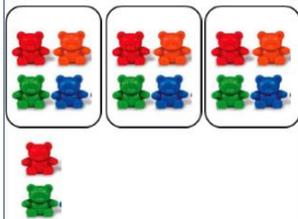
$$40 \div 5$$

Ask "How many 5s in 40?"



Division with remainders (Two digit by 1 digit)

Divide objects between groups and see how much is left over e.g. $14 \div 3 =$



Use equipment such as place value counters

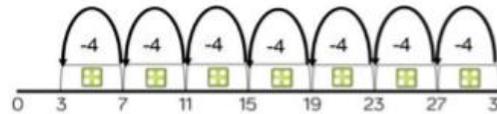
Step 1 – Build the number and show the groups on the place value chart.

Step 2 Share the tens

Step 3 Exchange the spare tens into ones and share the ones

94 - 4 =		94 - 4 =		94 - 4 = 23 r 2	
T	O	T	O	T	O
10 10 10 10	4 4 4 4	10 10	4 4 4 4	10 10 10 10	4 4 4 4 2 2
		1 ten left - exchange into ones		2 ones remaining	

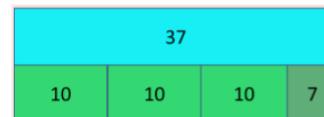
Children use number lines alongside equipment e.g. $31 \div 4$
How many groups of 4 have you subtracted? How many are remaining?



Draw dots and group them to divide an amount and clearly show a remainder.



Use bar models to show division with remainders.



Complete written divisions and show the remainder using r.

$$29 \div 8 = 3 \text{ REMAINDER } 5$$

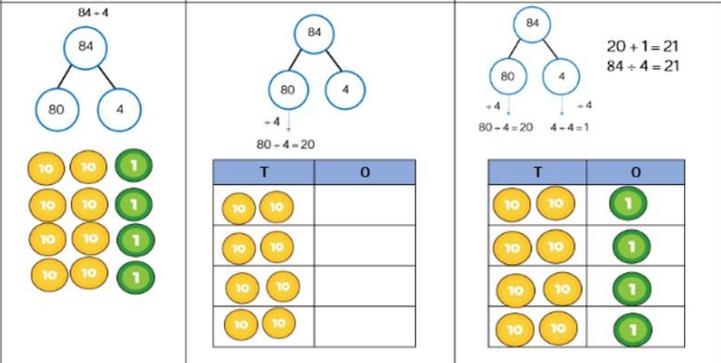
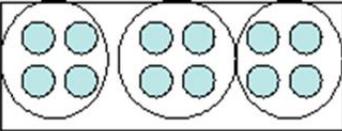
↑
↑
↑
↑

dividend
divisor
quotient
remainder

Division Year 4 statutory requirement:

Key Vocabulary:
 share, share equally one each, two each, three each... group in pairs, threes... tens equal groups of divide, division, divided by, divided into,

Year 4 statutory requirement: Note - there isn't a statutory objective for division. However, Y4 statutory multiplication objectives are to (1) recall multiplication and division facts for multiplication tables up to 12×12 and (2) multiply two-digit and three-digit numbers by a one-digit number using formal written layout so we will build on the connections between multiplication and division.

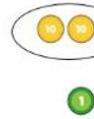
Objective & Strategy	Concrete	Pictorial	Abstract
<p>2 digit number divided by 1 digit - Share into equal groups (no remainders)</p> <p>Children build on their knowledge of dividing a two-digit number by a one-digit number from Year 3 by sharing into equal groups.</p>	<p>Children use place value counters to represent a calculation</p> <p>Step 1 – Build the number and show the groups on the place value chart. Step 2 Share the tens Step 3 Share the ones</p>  <p>They use the same approach but exchange e.g.</p>	<p>Children continue to draw their own diagrams with dots or circles to help them divide numbers into equal groups.</p> 	<p>Limit numbers to NO remainders in the answer OR carried (each digit must be a multiple of the divisor).</p> $\begin{array}{r} 32 \\ 3 \overline{) 96} \end{array}$

$91 \div 7 = 13$

1. Share the tens

T	O
10	1 1 1 1
10	1 1 1 1
10	1 1 1 1
10	1 1 1 1
10	1 1 1 1
10	1 1 1 1
10	1 1 1 1
10	1 1 1 1

2. Two tens left over



3. Exchange for 20 ones



4. Share the ones

2 digit number divided by 1 digit - Share into equal groups (with remainders)

Children use place value counters to represent a calculation
 Step 1 – Build the number and show the groups on the place value chart.
 Step 2 Share the tens
 Step 3 Share the ones

$87 \div 4$

$80 \div 4 = 20$

$7 \div 4 = 1 \text{ r } 3$

$20 + 1 \text{ r } 3 = 21 \text{ r } 3$
 $87 \div 4 = 21 \text{ r } 3$

They use the same approach but exchange

$77 \div 3 = 24 \text{ r } 1$

T	O
10 10	1 1 1 1
10 10	1 1 1 1
10 10	1 1 1 1

1 left over



Exchange for ten ones and share

e.g.

Children continue to draw their own diagrams with dots or circles to help them share numbers into equal groups.



Limit numbers to no remainders in the final answer, but with remainders occurring within the calculation.

$$4 \overline{) 732} \begin{matrix} 18 \\ \underline{72} \\ 10 \\ \underline{8} \\ 2 \end{matrix}$$

